Application of the PGD and DEI methods to solve a Non-Linear Magnetostatic Problem coupled with the Circuit Equations

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Among the model order reduction techniques, the Proper Generalized Decomposition (PGD) has shown its efficiency to solve static and quasistatic problems in the time domain. However, the introduction of nonlinearity due for example to ferromagnetic material has never been addressed. In this communication, the PGD technique combined with the Discrete Empirical Interpolation Method is applied to solve a non-linear problem in magnetostatic coupled with the circuit equations.

***Index Terms*— Non-linear magnetostatic problem, proper generalized decomposition, discrete empirical interpolation method**

I. Introduction

To reduce the computation time of time-dependent numerical models, Model Order Reduction (MOR) methods have been developed and presented in the literature. These methods consist in searching a solution in a subspace of the approximation space of the full numerical model. They have been mainly used to solve problems in mechanics. In this context, the Proper Generalized Decomposition (PGD) method has been developed since the early 2000's and is more and more applied. In the case of systems of partial differential equations in the time domain, the PGD method consists to approximate the solution by a sum of functions separable in time and space [1][2][3], so-called modes. Each mode is determined by an iterative procedure and depends on the previous modes. In the case of non-linear problems, the MOR methods are not so efficient than in the linear case, due to the computation cost of the non-linear terms. In fact, the calculation of the non-linear matrices of the reduced model requires the calculation of the non-linear matrices of the full model. To circumvent this issue, the Discrete Empirical Interpolation Method (DEIM) method can be used [4]. This method consists in interpolating the non-linear matrices of the full model by calculating only some of their entries. In the literature, the PGD approach has been combined with the DEIM in order to solve a thermal problem with a quadratic nonlinearity [5] but until now only linear problems have been solved with the PGD in computational electromagnetics [3].

In this communication, the PGD-DEIM approach is applied to solve a non-linear magnetostatic problem coupled with an external electric circuit using the vector potential formulation. The non-linearity of the ferromagnetic material is taken into account. A single phase transformer is studied with the proposed technique. The results obtained with the reduced model are compared in terms of accuracy and computation time with the full model.

II. PGD-DEIM model of Non-linear magnetostatic problem coupled with electric circuit

To solve the magnetostatic problem, the vector potential $\mathbf{A}$ is used, $\mathbf{B}(\mathbf{x},t)=\mathbf{curl}\mathbf{A}(\mathbf{x},t)$. To take into account the non-linear behavior of the ferromagnetic material, the magnetic field $\mathbf{H}(\mathbf{x},t)$ can be expressed under the form $\mathbf{H}(\mathbf{x},t)=\nu_{fp}\mathbf{B}(\mathbf{x},t)+\mathbf{H}_{fp}(\mathbf{B}(\mathbf{x},t))$ with ν_{fp} an arbitrary constant and $\mathbf{H}_{fp}(\mathbf{B}(\mathbf{x},t))$ a virtual magnetisation. Then, the equations to solve on $D\times[0,T]$ when accounting for the circuit coupling are

$$\mathbf{curl}(\nu_{fp}\mathbf{curl}\mathbf{A}(\mathbf{x},t)) - \mathbf{N}(\mathbf{x})i(t) = -\mathbf{curl}(\mathbf{H}_{fp}(\mathbf{A}(\mathbf{x},t))) \qquad (1)$$

$$\frac{d}{dt}\int_D \mathbf{A}(\mathbf{x},t).\mathbf{N}(\mathbf{x})dD + Ri(t) = v(t) \qquad (2)$$

with $\mathbf{N}$, v(t), i(t) and R the unit current density vector, the voltage, the current and the resistance associated with a stranded inductor. To solve (1) and (2), the PGD approach can be used. The vector potential $\mathbf{A}$ is then approximated by a separated representation of space and time functions,

$$\mathbf{A}_n(\mathbf{x},t) \approx \sum_{i=1}^{n} \mathbf{R}_i(\mathbf{x})S_i(t) \qquad (3)$$

with $\mathbf{x}\in D$, $t\in[0,T]$. To compute the functions $\mathbf{R}_n(\mathbf{x})$, $S_n(t)$ and i(t), an iterative procedure is used. At the n^{th} iteration, the solution is $\mathbf{A}_n(\mathbf{x},t) = \mathbf{R}_n(\mathbf{x})S_n(t) + \mathbf{A}_{n-1}(\mathbf{x},t)$ with $\mathbf{R}_n(\mathbf{x})$ and $S_n(t)$ the unknown functions. The current i(t) is recalculated at each iteration. To calculate $\mathbf{R}_n(\mathbf{x})$, $S_n(t)$ and i(t), two sets of equations deduced from weak forms (1) and (2), are solved iteratively [3]. First, we suppose that $S_n(t)$ and i(t) are known. According to (1), it can be shown that the function $\mathbf{R}_n(\mathbf{x})$ is the solution of the following equation

$$A_S\,\mathbf{curl}(\nu_{fp}\,\mathbf{curl}\,\mathbf{R}_n(\mathbf{x})) = \mathbf{F}_{Si} + \mathbf{F}_R(\mathbf{A}_{n-1}) + \mathbf{H}_{RNL}(\mathbf{A}_n) \qquad (4)$$

with A_S a constant term depending on $S_n(t)$, $\mathbf{F}_{Si}$ and $\mathbf{F}_R$ on ($S_n(t)$, i(t)) and the previous mode $\mathbf{A}_{n-1}$ and $\mathbf{H}_{RNL}$ the non-linear term. Secondly, to compute $S_n(t)$ and i(t), we assume that the function $\mathbf{R}_n(\mathbf{x})$ is known. According to (1) and (2), $S_n(t)$ and i(t) are the solutions of the following equations

$$A_R\,S_n(t) - C_R i(t) = \mathbf{F}_S(\mathbf{A}_{n-1}) + \mathbf{H}_{SNL}(\mathbf{A}_n) \qquad (5)$$

$$R\,i(t) + C_R\frac{dS_n(t)}{dt} = F_i(v(t)) \qquad (6)$$

with A_R and C_R terms depending on $\mathbf{R}_n(\mathbf{x})$, $\mathbf{F}_S$ and F_i depending on the previous mode $\mathbf{A}_{n-1}$ and the voltage v(t) and $\mathbf{H}_{SNL}$ the non-linear term. These two steps are repeated until convergence of the functions $\mathbf{R}_n(\mathbf{x})$, $S_n(t)$ and i(t). To reduce the computation time of PGD approximation, two methods has been used. The first method is the DEIM which allows to reduce the computation cost of the non-linear terms $\mathbf{H}_{RNL}$ and $\mathbf{H}_{SNL}$ [4][5][6]. After each computation of the functions $\mathbf{R}_n(\mathbf{x})$ and $S_n(t)$, the DEIM algorithm selects a small number N_{DEIM} of degrees of freedom depending on the non-linear behaviour law and to the approximated solution $\mathbf{A}_n(\mathbf{x},t)$. Then, to determine the vectors $\mathbf{H}_{RNL}$ and $\mathbf{H}_{SNL}$ in (4) and (5), the N_{DEIM} non-linear terms are computed and the other terms are interpolated. After each convergence of the iterative method, we obtain the approximated solution $\mathbf{A}_n(\mathbf{x},t)$ and the current i(t) of equations (1) and (2). The second method consist in recalculating of all functions depending to the time $S_i(t)$ in order to reduce the number of modes [2]. The functions $S_i(t)$ and i(t) are recalculated by projecting the residual function of (1) on the space spanned by the functions $\mathbf{R}_i(\mathbf{x})$.

III. Application

A 3D magnetostatic example, made of a single phase EI transformer at no load supplied at 50Hz with a sinusoidal voltage, is studied. Due to the symmetry, only one eighth of the transformer is modeled (Fig. 1). The non-linear magnetic behavior of the iron core is considered. The 3D spatial mesh is made of 12659 nodes and 67177 tetrahedrons. The time interval of simulation is fixed at [0;0.5s] in order to obtain an evolution of the current close to the steady state at t=0.5s. The time step is fixed at 0.5ms, the number of time steps is equal to 1000.

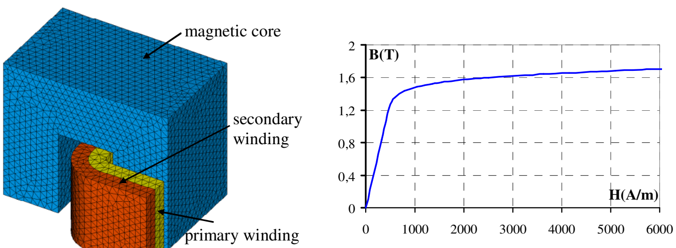

Fig. 1. Application example (a: geometry, b: non-linear curve)

The DEIM approach selects N_{DEIM} of degrees of freedom used to compute the non-linear terms $\mathbf{H}_{RNL}$ and $\mathbf{H}_{SNL}$ from the solutions at several time steps. N_{DEIM} is updated after each computation of a mode. Figure 2 presents the edges, which are related to the degrees of freedom, selected by the DEIM and obtained at the last iteration. These are located in the saturated area. Figure 3 presents the relative error of the current obtained from the PGD and full models versus the number of modes. The error decreases rapidly when the number of modes increases. Figure 4 presents the evolution of the current for the transient state obtained from the full model and the PGD model with seven modes. We can see a good agreement between the two solutions. When the PGD solution is approximated by seven modes, the relative error of the current is of about 1.7% and the speed up between the full and PGD modes is equal to 3. On this example, we can see that the PGD enables to determine quickly the global quantity with a low number of modes and with a computation time significantly reduced.

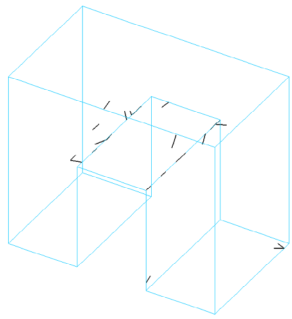

Fig. 2. Edges selected by the DEIM algorithm

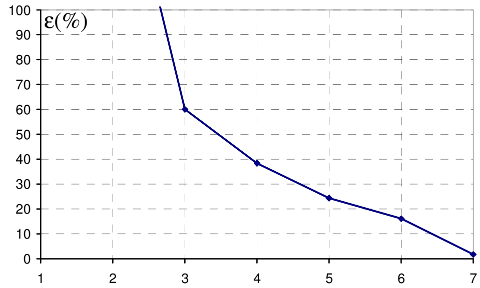

Fig. 3. Relative error of the current obtained from the PGD and full models versus the number of mode

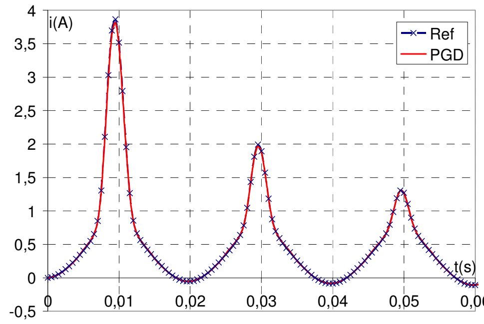

Fig. 4. Evolution of the current obtained from the full and PGD models with seven modes for the transient state

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